DHAKSHINA EDUCATIONAL SERVICES PVT.LTD.

XII CBSE MATHS 100 MARKS 3 Hrs

Q.No. 1 to 5 carry each 2 M, Q.No. 6 to 17 carry each 4 M, Q.No. 18 to 24 carry each 6 M

1. Let L be the set of all the lines in a plane and R be the relation in L defined as R $\{(l_1, l_2) : l_1$ is perpendicular to $l_2\}$. Show that R is symmetric but neither reflexive nor transitive.

2. Show that the function f : N → N given by $f(x) = \begin{cases} x+1, & \text{if x is odd} \\ x-1, & \text{if x is even} \end{cases}$ is both one-one and onto.

3. If $y = \tan^{-1}\left[\frac{a\cos x + b\sin x}{b\cos x - a\sin x}\right]$, find $\frac{dy}{dx}$

4. Evaluate: $\int \frac{1}{x^2}\cos\left(\frac{1}{x}\right)dx$ (OR) Evaluate $\int \frac{x^2-1}{x^4+1}dx$

5. Examine the continuity for the function $f(x) = |x - 5|$.

6. Solve the differential equation: $\frac{dy}{dx} = \sin^3 x \cos^2 x + xe^x$.

7. Find the area of a triangle whose vertices are (3, -1, 2), (1, -1, -3), (4, -3, 1).

8. The mean and variance of a binomial distribution are 10 and $\frac{5}{3}$ respectively. Find P ($X \geq 1$).

9. In an examination an examinee either guesses or copies or knows the answer of multiple choice questions with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answered it.

10. Show that, $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$ (OR)

Find the value of $\tan\left[\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2} + \frac{1}{2}\cos^{-1}\frac{1-y^2}{1+y^2}\right]$

11. For what value of k, the points (5, 5), (k, 1) and (11, 7) are collinear? What figure do you get when k = 1? Find its area using determinants.

12. If $y = \sin^{-1}(x^2\sqrt{1-x^2} + x\sqrt{1-x^4})$, then prove that $\frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}} + \frac{1}{\sqrt{1-x^2}}$

13. For what values of x, the function $f(x) = \frac{x}{2} + \frac{2}{x}$, $x \neq 0$ is increasing or decreasing on R.

14. Prove that $\int_0^{\pi/4} \log(1+\tan x)dx = \frac{\pi}{8}\log 2$.

15. Solve the differential equation $y - x\frac{dy}{dx} = a\left(y^2 + x^2\frac{dy}{dx}\right)$; y(a) = a

16. If $|a| = 3, |b| = 4, |c| = 5$, such that each is perpendicular to the sum of the other two, find $|a+b+c|$.

17. Find the distance between the parallel planes $4x + 4y + 4z + 15 = 0$ and $2x + 2y + 2z + 9 = 0$

18. Solve : $\int 5^{5^{5^x}}.5^{5^x}.5^x \, dx$

19. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the mean and the variance of the number of kings. (OR)
A pair of dice is thrown 7 times. If getting a total of 7 is considered a success, what is the probability of at least 6 successes.

20. If $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ prove that $A^{-1} = A^2 - 6A + 11I$. (OR)

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the following system of linear equations:

$2x - 3y + 5z = 11$; $3x + 2y - 4z = -5$; $x + y - 2z = -3$.

21. A cylinder is such that the sum of its height and circumference of its base is 10 meters. Find the maximum volume of the cylinder.

22. Draw a rough sketch of the region: $\{(x, y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$ and find the enclosed area, using method of integration.

23. A variable plane which remains at a constant distance 3p from origin cuts the coordinate axes at A, B and C. Show that the locus of centroid of triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$

24. A farmer decides to plant up to 10 hectares with cabbages and potatoes. He decides to grow at least 2, but not more than 8 hectares of cabbage and at least one but not more than 6 hectares of potatoes. If he can make a profit of Rs. 1,500 per hectare on cabbage and Rs. 2,000 per hectare on potatoes, how should he plan his farming so as to get the maximum profit, assuming the total yield that he gets is sold. (OR)

A manufacturer is trying to decide on the product quantities of two products — tables and chairs. There are 98 units of material and 80 labour-hours available. Each table requires 7 units of material and 10 labour hours, while each chair requires 14 units of material and 8 labour-hours per chair. The profit on a table and a chair is Rs. 25 and Rs. 20 respectively. How many tables and chairs should be produced to have maximum profit?