DHAKSHINA EDUCATIONAL SERVICES PVT. LTD.

XII STATE MATHS 200MARKS 3 Hrs

SECTION – A 40 x 1 = 40

1. If A is a matrix of order 3, then det (KA) is
(a) k^3 det (A) (b) k^2 det (A) (c) k det (A) (d) det A
2. The system of equations ax + y + z = 0; x + by + z = 0; x + y + cz = 0 has a non-trivial solution then $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=$
(a) 1 (b) 2 (c) –1 (d) 0
3. If A = $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ then (adj A) A =
(a) $\begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
4. In the system of 3 linear equations with three unknowns, if $\Delta = 0$, $\Delta_x = 0$, $\Delta_y = 0$, $\Delta_z = 0$ and atleast one 2 x 2 minor of $\Delta \neq 0$ then the system is
(a) consistent (b) inconsistent (c) consistent and the system reduces to two equations
(d) consistent and system reduces to a single equation.
5. The work done by the force $\vec{F} = \vec{i} + \vec{j} + \vec{k}$ acting on a particle, if the particle is displaced from A (3, 3, 3) to the point B(4, 4, 4) is
(a) 2 units (b) 3 units (c) 4 units (d) 7 units
6. If $\vec{a}, \vec{b}, \vec{c}$ are a right handed triad of mutually perpendicular vectors of magnitude a, b, c then the value of $[\vec{a}\ \vec{b}\ \vec{c}]$ is
(a) $a^2\ b^2\ c^2$ (b) 0 (c) ½ abc (d) abc
7. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then
(a) $\vec{a}$ is parallel to $\vec{b}$ (b) $\vec{a}$ is perpendicular to $\vec{b}$
(c) $|\vec{a}| = |\vec{b}|$ (d) $\vec{a}$ and $\vec{b}$ are unit vectors
8. If $\vec{u} = \vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b})$ then
(a) $\vec{u}$ is a unit vector (b) $\vec{u} = \vec{a} + \vec{b} + \vec{c}$ (c) $\vec{u} = \vec{0}$ (d) $\vec{u} \neq \vec{0}$
9. The length of the perpendicular from the origin to the plane $\vec{r} \cdot (3\vec{i} + 4\vec{j} + 12\vec{k}) = 26$ is
(a) 26 (b) $\frac{26}{169}$ (c) 2 (d) ½
10. The non-parametric vector equation of a plane passing through a point whose P . V is $\vec{a}$ and parallel to $\vec{u}$ and $\vec{v}$ is
(a) $[\vec{r} - \vec{a}\ \vec{u}\ \vec{v}] = 0$ (b) $[\vec{r}\ \vec{u}\ \vec{v}] = 0$ (c) $[\vec{r}\ \vec{a}\ \vec{u} \times \vec{v}] = 0$ (d) $[\vec{a}\ \vec{u}\ \vec{v}] = 0$
11. If A + iB = $(a_1 + ib_1)(a_2 + ib_2)(a_3 + ib_3)$ then $A^2 + B^2$ is
(a) $a_1^2 + b_1^2 + a_2^2 + b_2^2 + a_3^2 + b_3^2$ (b) $(a_1 + a_2 + a_3)^2 + (b_1 + b_2 + b_3)^2$
(c) $(a_1^2 + b_1^2)(a_2^2 + b_2^2)(a_3^2 + b_3^2)$ (d) $(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$
12. $\frac{1+e^{-i\theta}}{1+e^{i\theta}} =$
(a) cos θ + i sin θ (b) cos θ - i sin θ (c) sin θ - i cos θ (d) sin θ + i cos θ
13. If – i + 3 is a root of $x^2 – 6x + k = 0$ then the value of k is
(a) 5 (b) $\sqrt{5}$ (c) $\sqrt{10}$ (d) 10
14. Which of the following statements is correct?
(a) negative complex numbers exist (b) order relation does not exist in real numbers
(c) order relation exist in complex numbers (d) (1 + i) > (3 – 2i) is meaningless
15. The line 4x + 2y = c is a tangent to the parabola $y^2 = 16x$ then c is
(a) –1 (b) –2 (c) 4 (d) – 4
16. The length of the latus rectum of the parabola $y^2 – 4x + 4y + 8 = 0$ is
(a) 8 (b) 6 (c) 4 (d) 2
17. The asymptotes of the hyperbola $36y^2 – 25x^2 + 900 = 0$ are
(a) $y = \pm\frac{6}{5}x$ (b) $y = \pm\frac{5}{6}x$ (c) $y = \pm\frac{36}{25}x$ (d) $y = \pm\frac{25}{36}x$
18. The equations of the major and minor axes of $4x^2 + 3y^2 = 12$ are

(a) $x = \sqrt{3}, y = 2$ (b) x = 0, y = 0 (c) $x = -\sqrt{3}, y = -2$ (d) y = 0, x = 0

19. The slope of the tangent to the curve $y = 3x^2 + 3 \sin x$ at x = 0 is
(a) 3 (b) 2 (c) 1 (d) – 1
20. Which of the following function is increasing in (0, ∞)
(a) e^x (b) 1/x (c) $-x^2$ (d) x^{-2}
21. The point of inflexion of the curve $y = x^4$ is at
(a) x = 0 (b) x = 3 (c) x = 12 (d) nowhere
22. If $\lim\limits_{x \to a} g(x) = b$ and f is continuous at x = b then
(a) $\lim\limits_{x \to a} g(f(x)) = f\left(\lim\limits_{x \to a} g(x)\right)$ (b) $\lim\limits_{x \to a} f(g(x)) = f\left(\lim\limits_{x \to a} g(x)\right)$
(c) $\lim\limits_{x \to a} f(g(x)) = g\left(\lim\limits_{x \to a} f(x)\right)$ (d) $\lim\limits_{x \to a} f(g(x)) \neq f\left(\lim\limits_{x \to a} g(x)\right)$
23. If $u = \sin^{-1}\left(\frac{x^4 + y^4}{x^2 + y^2}\right)$ and f = sin u then f is a homogeneous function of degree
(a) 0 (b) 1 (c) 2 (d) 4
24. The curve $ay^2 = x^2 (3a - x)$ cuts the y – axis at
(a) x = - 3a, x = 0 (b) x = 0, x = 3a (c) x = 0, x = a (d) x = 0
25. The length of the arc of the curve $x^{2/3} + y^{2/3} = 4$ is
(a) 48 (b) 24 (c) 12 (d) 96
26. The volume generated by rotating the triangle with vertices at (0, 0), (3, 0) and (3, 3) about x – axis is
(a) 18π (b) 2π (c) 36π (d) 9π
27. The area bounded by the line y = x, the x – axis, the ordinates x = 1, x = 2 is
(a) 3/2 (b) 5/2 (c) ½ (d) 7/2
28. The surface area obtained by revolving the area bounded by the curve y = f(x), the two ordinates x = a, x = b and x – axis is
(a) $\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$ (b) $\int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dx$ (c) $2\pi \int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$ (d) $2\pi \int_a^b y\sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dx$
29. The differential equation satisfied by all the straight lines in xy plane is
(a) $\frac{dy}{dx} = a$ constant (b) $\frac{d^2y}{dx^2} = 0$ (c) $y + \frac{dy}{dx} = 0$ (x) $\frac{d^2y}{dx^2} + y = 0$
30. The integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$ is
(a) $\int Pdx$ (b) $\int Qdx$ (c) $e^{\int Qdx}$ (d) $e^{\int Pdx}$
31. The P.I. of $(3D^2 + D - 14)y = 13e^{2x}$ is
(a) $26x\, e^{2x}$ (b) $13\, x\, e^{2x}$ (c) $x\, e^{2x}$ (d) $x^2/2\, e^{2x}$
32. The order and degree of the differential equation are $\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^5 + y = 7$
(a) 3, 1 (b) 1, 3 (c) 3, 5 (d) 2, 3
33. If a compound statement is made up of three simple statements, ten the number of rows in the truth table is
(a) 8 (b) 6 (c) 4 (d) 2
34. A monoid becomes a group if it also satisfies the
(a) closure axiom (b) associative axiom (c) identity axiom (d) inverse axiom
35. Which of the following is correct?
(a) An element of a group can have more than one inverse.
(b) If every element of a group is its own inverse, then the group is abelian.
(c) The set of all 2 x 2 real matrices forms a group under matrix multiplication.
(d) $(a * b)^{-1}$ $a^{-1} * b^{-1}$ for all a, b ∈ G
36. Which of the following are not statements?
(i) Three plus four is eight (ii) The sun is a planet
(iii) Switch on the light (iv) Where are you going ?
(a) (i), (ii) (b) (ii), (iii) (c) (iii) and (iv) (d) (iv) only
37. $\mu_2 = 20$, $\mu_2' = 276$ for a discrete random variable X. Then the mean of the random variable X is
(a) 16 (b) 5 (c) 2 (d) 1

38. The mean of a binomial distribution is 5 and its standard deviation is 2. Then the value of n and p are

(a) $\left(\frac{4}{5}, 25\right)$ (b) $\left(25, \frac{4}{5}\right)$ (c) $\left(\frac{1}{5}, 25\right)$ (d) $\left(25, \frac{1}{5}\right)$

39. A box contains 6 red and 4 white balls. If 3 balls are drawn at random, the probability of getting 2 white balls is

(a) $\frac{1}{20}$ (b) $\frac{18}{125}$ (c) $\frac{4}{25}$ (d) $\frac{3}{10}$

40. For a standard normal distribution the mean and variance are

(a) μ, σ^2 (b) μ, σ (c) 0, 1 (d) 1, 1

SECTION – B Answer any 10 55 is compulsory 10 x 6 = 60

41. Solve the following non-homogeneous system of linear equations by determinant method
$4x + 5y = 9$; $8x + 10y = 18$

42. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$.

43. Find the coordinates of the centre and the radius of the sphere whose vector equation is
$r^2 - r.(8i - 6j + 10k) - 50 = 0$

44. (i) Show that vectors a, b, c are coplanar if and only if $a + b, b + c, c + a$ are coplanar.

(ii) Find the angle between the following lines. $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-4}{6}$ and
$x + 1 = \frac{y+2}{2} = \frac{z-4}{2}$

45. P represents the variable complex number z. Find the locus of P if $\text{Re}\left(\frac{z+1}{z-i}\right) = 0$

46. Prove that the tangent at any point to the rectangular hyperbola forms with the asymptotes a triangle of constant area.

47. (i) Verify Rolle's theorem for the following: $f(x) = x^3 - 3x + 3$, $0 \le x \le 1$

(ii) Evaluate : $\lim_{x \to \infty} \frac{x^2}{e^x}$

48. Find the intervals of concavity and the points of inflection of the function $f(x) = 2x^3 + 5x^2 - 4x$

49. If $w = x + 2y + z^2$ and $x = \cos t$; $y = \sin t$; $z = t$, find $\frac{dw}{dt}$

50. Evaluate $\int_0^{\pi/2} \log(\tan x)dx$

51. Solve : $\frac{dy}{dx} + xy = x$

52. Show that $\sim (p \vee q) \equiv (\sim p) \wedge (\sim q)$

53. Show that $\sim(p \wedge q) \equiv ((\sim p) \vee (\sim q))$

54. Find the mean and variance of the distribution $f(x) = \begin{cases} 3e^{-3x} & , 0 < x < \infty \\ 0 & , elsewhere \end{cases}$

55. (a) Marks in an aptitude test given to 800 students of a school was found to be normally distributed. 10% of the students scored below 40 marks and 10% of the students scored above 90 marks. Find the number of students scored between 40 and 90.

(OR)

(b) Prove that the triangle formed by the points representing the complex numbers (10 + 8i), (-2 + 4i) and (-11 + 31i) on the Argand plane is right angled.

SECTION – C Answer any 10 70 is compulsory (10 x 10 = 100)

56. Discuss the solutions of the system of equations for all values of λ by using rank.
$x + y + z = 2$, $2x + y - 2z = 2$, $\lambda x + y + 4z = 2$

57. Find the vector and Cartesian equation of the plane passing through a point (-1, -2, 1) and perpendicular to two planes $x + 2y + 4z + 7 = 0$ and $2x - y + 3z + 3 = 0$

58. If α and β are the roots of $x^2 - 2x + 2 = 0$ and $\cot \theta = y + 1$, show that
$\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = \frac{\sin n\theta}{\sin^n \theta}$

59. Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below

the line of the pipe, the flow of water has curved outward 3 m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground/

60. A kho-kho player in a practice session while running realizes that the sum of the distances from the two kho-kho poles from him is always 8 m. Find the equation of the path traced by him if the distance between the poles is 6m.

61. A farmer has 2400 feet of fencing and want to fence of a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

62. At noon, ship A is 100 km west of ship B. Ship A is sailing east at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4.00 p.m.

63. Trace the curve $y = x^3 + 1$.

64. Find the area bounded by the curve $y = x^3$ and the line $y = x$.

65. prove that the curved surface area of a sphere of radius r intercepted between two parallel planes at a distance a and b from the centre of the sphere is $2\pi r\,(b - a)$ and hence deduct the surface area of the sphere $(b > a)$.

66. A bank pays interest by continuous compounding, that is by treating the interest rate as the instantaneous rate of change of principal. Suppose in an account interest accrues at 8% per year compounded continuously. Calculate the percentage increase in such an account over one year. [Take $e^{.08} \approx 1.0833$]

67. Solve $(D^2 - 1)y = \cos 2x - 2 \sin 2x$.

68. Show that $\left\{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix}\right\}$ where $\omega^3 = 1, \omega \neq 1$ form a group with respect to matrix multiplication.

69. The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers find approximately the number of drivers with (i) no accident in a year (ii) more than 3 accidents in a year [$e^{-3} = 0.0498$].

70. (a) Find the eccentricity, centre, foci and vertices of the following hyperbola and draw its diagram $x^2 - 3y^2 + 6x + 6y + 18 = 0$

(OR)

(b) If $a = 2i + 3j - k, \quad b = -2i + 5k, \quad c = j - 3k$ verify that $a \times (b \times c) = (a \cdot c)\, b - (a \cdot b)\, c$